How Do Sets of Color-Matching Functions Differ?

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Introduction

There are currently three "standard" sets of color-matching functions (CMFs) characterizing 2° foveal color vision. These are the 1931 CIE, the Judd (1951) version of the CIE and the Stiles (1955) functions. The 1931 CIE used spectral coefficients collected by Wright (1929) and Guild (1931). Wright obtained spectral coefficients for the NPL "white", a white of about 4800 K. The two data sets were transformed to the same primaries, those of the RGB observer (Wyszecki and Stiles, 1967) but normalized to the NPL white. The data were then converted to color matching functions by Wright's method which assumed that the luminosity function was a linear combination of the CMFs. The 1924 CIE relative photopic luminous efficiency factor was used as the luminosity function. The data were subsequently transformed to the RGB and XYZ primary systems. When it became evident that the CIE 1924 standard luminous efficiency function underestimated the luminosity of many observers at wavelengths below 460 nm, Judd (1951) derived revised luminosity values based on data obtained in several laboratories during 1945–1950. These new values were combined with the data of Gibson and Tyndall (1923) on which the 1924 standard observer had in part been based. Judd (1951) then used these new luminosity values to revise the color matching functions.

The color matching functions of Stiles (1955) were obtained with the NPL trichromator and represent pilot data for ten observers. The functions are reported for an equal-energy spectrum. Subsequently, a calibration correction was reported in Stiles and Burch (1959).

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The CIE observer is used by color theorists to derive sets of color fundamentals or spectral sensitivities of presumed cone photoreceptors. Concern that the CIE luminosity was not representative of an "average" observer led naturally to the use of the Judd (1951) revision (e.g., Thomson and Wright, 1953; Vos and Walraven, 1971; Smith and Pokorny, 1975). Recently, Estévez (1979) has suggested that the Stiles (1955) pilot data may be a better data set for color-modelling purposes. Data from individuals are not available for either the CIE or the Stiles (1955) observers. The CIE observer that combines photometric and colorimetric properties has the advantage to the color theorist that it allows prediction of luminosity by a linear combination of cone fundamentals. This operation is justified to the extent that heterochromatic luminances are additive (e.g., Dresler, 1953; Wagner and Boynton, 1972; Eisner and MacLeod, 1981). However, criticism may be directed at the fact that the spectral coefficients were characteristic of different observers than the luminosity values. A major disadvantage of the Stiles (1955) data set is that the luminance level of the different test wavelengths varied considerably owing to the use of an equal-energy spectrum. There is the possibility that the data are contaminated by rod intrusion (see, for example, Estévez, 1979).

In this communication we ask the question, "To what extent do the three data sets differ?" If the data sets differ only by differences in ocular media transmission then the three sets are essentially equivalent for color-modelling purposes. If more serious differences are found, some rational choice would have to be made as to the preferred set.

Evaluation of the Color-Matching Functions

Our first step was to evaluate differences in the three sets of color matching functions. Our null hypothesis was to assume that the three data sets differ only in pre-retinal screening factors.

The pre-retinal screening factors we allowed were the lens and the macular pigment. For lens we used the density spectrum, LN, tabulated by Wyszecki and Stiles (1967). Van Norren and Vos (1974) noted that this tabulation is consistent with the Boettner and Wolter (1962) data for total pre-retinal ocular media (i.e., cornea, lens and vitreous humor). Van Norren and Vos (1974) reviewed existing studies of the ocular media and recommended a density function for total ocular media that differed from the Wyszecki and Stiles (1967) tabulation only below 420 nm. For macular pigment we used the density spectrum, MP, tabulated by Wyszecki and Stiles (1967). This tabulation is similar to one suggested by Vos (1972) after an extensive review of the literature on macular pigment.

According to the null hypothesis, the ratio of two sets of CMFs will be equal to the ratio of the pre-retinal transmittances. If the null hypothesis is correct, this ratio of the transmittances, which will be in transmittance units, can be fit by the antilog of a linear combination of lens, LN, and macular pigment, MP. A set of color-matching functions, CMF(I), can then be adjusted to another set, CMF(II), by multiplying the first set by a pre-retinal screening factor, T^*.

$$\mathrm{CMF(II)} = \mathrm{CMF(I)}T^* \quad (1)$$
$$T^* = 10^{(K_1 LN + K_2 MP)} \quad (2)$$

We have chosen to work with T*, the inverse of the ratio of transmittances, so that the weights, K_1 and K_2, have a convenient interpretation. K_1 and K_2 will be proportional to the difference between observers in optical pathlength through the lens and macular pigments respectively.

This calculation may be made for each CMF individually and for an average of the three CMFs. The two sets of color matching functions must be expressed in the same primary and normalization system. The average may be a simple sum or a weighted average of the three primaries. We used equal energy CMFs in the Stiles primary system (15,500, 19,000 and 22,500 cm^{-1}) and a simple average of the three functions.

We made three comparisons: we adjusted the CIE to the Judd; we adjusted the Stiles to the Judd; and we adjusted the Stiles to the CIE. We calculated the logarithmic ratio for each comparison pair and found the values of K_1 and K_2 that minimized the square of the errors in the linear equation:

$$\log[\mathrm{CMF(II)/CMF(I)}] = K_1 LN + K_2 MP + \text{error} \quad (3)$$

The results are expressed in Table 1 and the three tabulations of T* are given in Table 2. In the comparison of the CIE to the Judd, the CIE observer showed more lens pigmentation but less macular pigment than the Judd observer.

TABLE 1 Differences in optical densities of lens and macular pigment between different sets of CMFs*

Set I	Set II	Lens		Macular pigment	
		K_1	O.D. difference at max.	K_2	O.D. difference at max.
CIE	Judd	0.611	0.733	−0.299	−0.148
Stiles	Judd	0.288	0.346	−0.085	−0.042
Stiles	CIE	−0.323	−0.387	0.214	0.106

* K_1 and K_2 are the coefficients derived in Equation 3.

Lens and macular pigment are expressed in density units at their maxima (i.e., 1.2 at 25,000 cm^{-1} for the lens and 0.495 at 21,700 cm^{-1} for the macular pigment.)

A positive number means that Set I had higher optical density than Set II.

A negative number means that Set I had lower optical density than Set II.

The Judd observer, as intended by the corrections, is characterized by a lower lens factor than the CIE observer. Additionally, Judd built slightly higher macular pigment into his revised observer. These calculations confirm a similar observation of Stiles (1955) concerning the Judd observer.

In the comparison of the Stiles to the Judd observer, the Stiles observer showed more lens and slightly less macular pigment. Finally, in the comparison of the Stiles observer to the CIE observer, the Stiles observer showed less lens but more macular pigment.

The values in Table 1 depend on the normalization of CMFs. We also checked the effect of WDW normalization. This normalization reduces the contribution of the "red" primary CMF to the unweighted average. The main trends of the data were the same but the derived values of lens and macular pigment differed slightly.

Figures 1 to 3show the set II CMFs (continuous lines) compared with the set I CMFs (symbols) multiplied by the factors T* given in Table 2 and renormalized at each primary. In each figure the symbols show minor deviations from the solid lines. Few of the differences appear systematic. There is no statement of interobserver variability for the three sets of CMFs. None of the deviations exceed the interobserver variability of the Stiles and Burch (1959) 10° data or the Viénot (1980) 10° data.

Our conclusion from this analysis is that the CIE, the Judd and the Stiles observers differ primarily in lens and macular pigmentation. Judd's correction was intended to do more than add inert pigments. However, the residual deviations are not great.

Fundamentals Derived from the CMFs

Given the conclusion that the three sets of CMFs are characterized by the same underlying set of cone visual photopigment spectra, we can now ask whether a candidate set of fundamentals can fit the CIE and Stiles observers. We chose to evaluate our set of König fundamentals (Smith and Pokorny, 1975) derived from the Judd (1951) observer. Identical results were obtained using the analytical functions described by Boynton and Wisowaty (1980).

Our first step was to transform the copunctal points characteristic of the Smith and Pokorny (1975) fundamentals to the Stiles primary system. These copunctal points are shown in Table 3. The solution equations for the fundamentals can be solved by inverting the matrix of the copunctal points. This solution gives arbitrary heights and the three fundamentals should be normalized. Table 3 shows the solution equations for the Judd observer normalized to unit sensitivity. Fundamentals based upon the same copunctal points but other color-matching functions are obtained using the weighted CMFs and normalizing. The solution equations for the CIE and Stiles observers are shown in Table 4. There are no free parameters in determining the shape of the

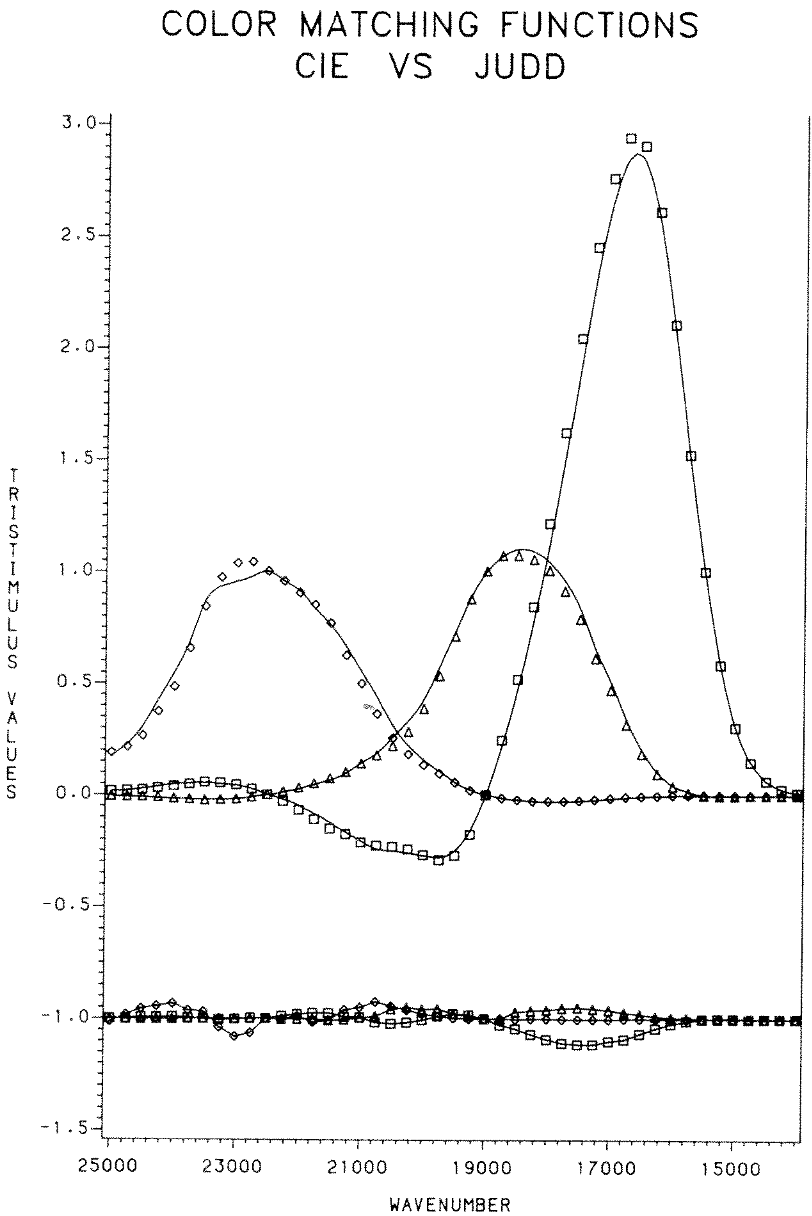

FIG. 1 The top panel shows the Judd CMFs as solid lines and the CIE CMFs adjusted for lens and macular transmission to the Judd CMFs as symbols. The primaries are P_1, 15000; P_2, 19000; P_3, 22500 wavenumber (cm^{-1}). The bottom panel shows the errors centered on -1.0. Symbols: □, $\bar{p}_1$; △ $\bar{p}_2$; ◇ $\bar{p}_3$.

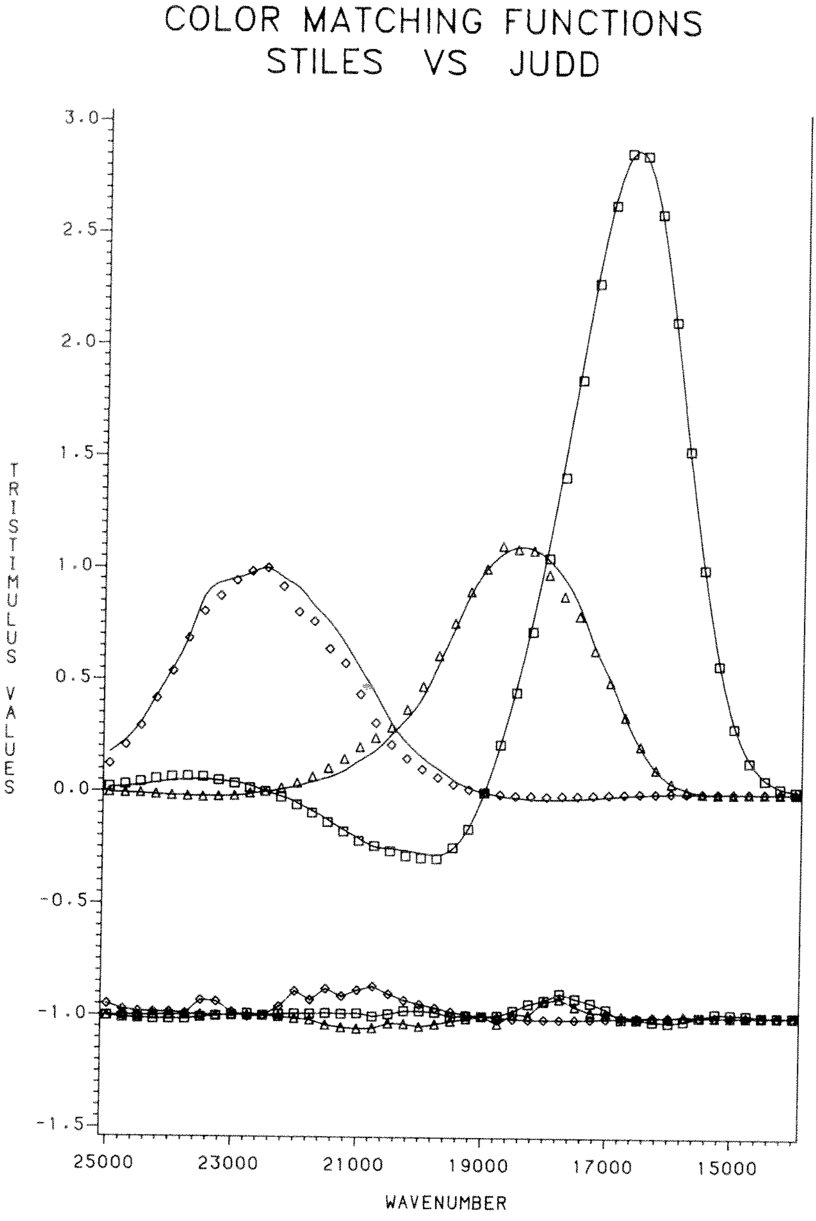

FIG. 2 Comparison of Judd [solid lines] and Stiles [symbols] CMFs. The Stiles CMFs were adjusted for lens and macular transmission to the Judd. Symbols: □, $\bar{p}_1$; △ $\bar{p}_2$; ◇ $\bar{p}_3$.

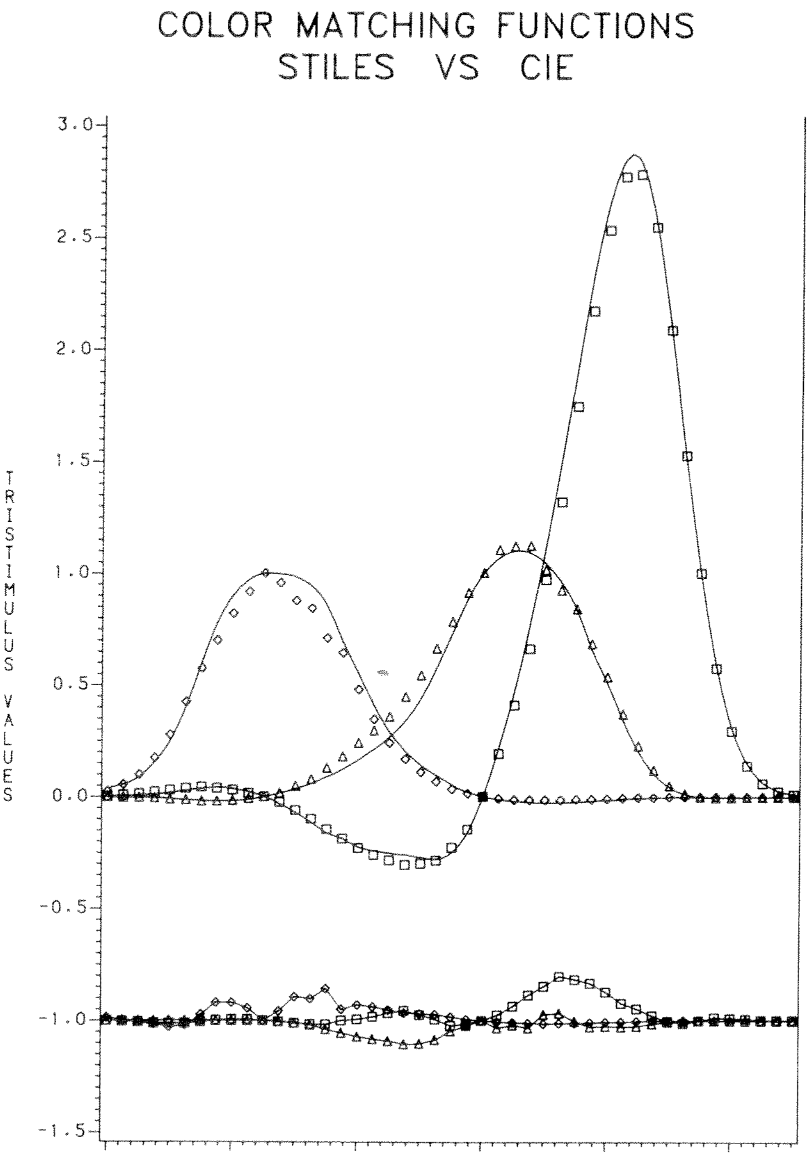

FIG. 3 Comparison of CIE [solid lines] and Stiles [symbols] CMFs. The Stiles CMFs were adjusted for lens and macular transmission to the CIE. Symbols: □, $\bar{p}_1$; △ $\bar{p}_2$; ◇ $\bar{p}_3$.

TABLE 2 Values of T* to minimize differences between color-matching functions

Wavenumber (cm^{-1})	CMF(I): CIE CMF(II): Judd	CMF(I): Stiles CMF(II): Judd	CMF(I): Stiles CMF(II): CIE
25000	5.0988	0.4277	2.1807
24750	3.9185	0.4934	1.9334
24500	3.0920	0.5613	1.7357
24250	2.4746	0.6347	1.5706
24000	2.0093	0.7145	1.4356
23750	1.6843	0.7900	1.3307
23500	1.4560	0.8580	1.2492
23250	1.2968	0.9139	1.1851
23000	1.1984	0.9546	1.1440
22750	1.1176	0.9926	1.1094
22500	1.0435	1.0354	1.0804
22250	1.0027	1.0591	1.0619
22000	0.9583	1.0896	1.0442
21750	0.9450	1.0984	1.0380
21500	0.9394	1.0987	1.0322
21250	0.9378	1.0958	1.0276
21000	0.9506	1.0830	1.0295
20750	0.9324	1.0948	1.0209
20500	0.9210	1.1013	1.0143
20250	0.9387	1.0842	1.0178
20000	0.9897	1.0419	1.0312
19750	1.0456	0.9997	1.0453
19500	1.0956	0.9649	1.0572
19250	1.1160	0.9504	1.0607
19000	1.1312	0.9394	1.0627
18750	1.1326	0.9367	1.0609
18500	1.0996	0.9511	1.0458
18250	1.0938	0.9537	1.0432
18000	1.0880	0.9564	1.0406
17750	1.0766	0.9618	1.0354
17500	1.0653	0.9671	1.0303
17250	1.0541	0.9725	1.0252
17000	1.0432	0.9779	1.0202
16750	1.0376	0.9807	1.0176
16500	1.0289	0.9851	1.0135
16250	1.0213	0.9889	1.0100
16000	1.0159	0.9917	1.0075
15750	1.0106	0.9944	1.0050
15500	1.0053	0.9972	1.0025
15250	1.0000	1.0000	1.0000
15000	1.0000	1.0000	1.0000
14750	1.0000	1.0000	1.0000
14500	1.0000	1.0000	1.0000
14250	1.0000	1.0000	1.0000
14000	1.0000	1.0000	1.0000

TABLE 3 Coefficients to derive Judd-based König fundamentals in Stiles primary system

Copunctal points:	p_1	p_2	p_3
Deuteranope	1.3741	−0.3868	0.0127
Protanope	1.0226	−0.0233	0.0070
Tritanope	0.0507	−0.0606	1.0099

Solution equations:

$$S_R = 0.2034\bar{p}_1 + 0.7239\bar{p}_2 + 0.0332\bar{p}_3$$
$$S_G = 0.0205\bar{p}_1 + 0.9022\bar{p}_2 + 0.0531\bar{p}_3$$
$$S_B = 0.0293\bar{p}_2 + 1.000\bar{p}_3$$

TABLE 4 Solution equations to derive fundamental sensitivities to the CIE and Stiles observers in the Stiles primary system

CIE observer

$$S_R = T^*(0.1870\bar{p}_1 + 0.6723\bar{p}_2 + 0.0309\bar{p}_3)$$
$$S_G = T^*(0.0187\bar{p}_1 + 0.8204\bar{p}_2 + 0.0483\bar{p}_3)$$
$$S_B = T^*(0.0302\bar{p}_2 + 0.9202\bar{p}_3)$$

T* is tabulated in Table 2 (Column 2)

Stiles observer

$$S_R = T^*(0.2067\bar{p}_1 + 0.7355\bar{p}_2 + 0.0338\bar{p}_3)$$
$$S_G = T^*(0.0193\bar{p}_1 + 0.8496\bar{p}_2 + 0.0500\bar{p}_3)$$
$$S_B = T^*(0.0304\bar{p}_2 + 0.9202\bar{p}_3)$$

T* is tabulated in Table 2 (Column 3)

fundamentals, the solution equations differ only in the setting of the peaks to unity. The resulting fundamentals are plotted in Fig. 4 and 5 and compared with the Smith and Pokorny (1975) fundamentals derived from the Judd observer. The fundamentals derived from the CIE observer show a good fit to those derived from the Judd observer although there is residual deviation of the B fundamental at short wavenumbers. The comparison of the fit of the fundamentals derived from the Stiles observer to those derived from the Judd observer shows some deviations, generally not exceeding 0.2 logarithmic units. For the R and G fundamentals the fits are good at short wavenumbers and the major deviations occur near 21,500 cm^{-1}. The B fundamental shows more serious deviations for wavenumbers below 18,500 cm^{-1}. Estévez (1979) pointed out that the difficulty with the B fundamental occurs because the spectrum locus in the chromaticity chart is not described by a straight line at short

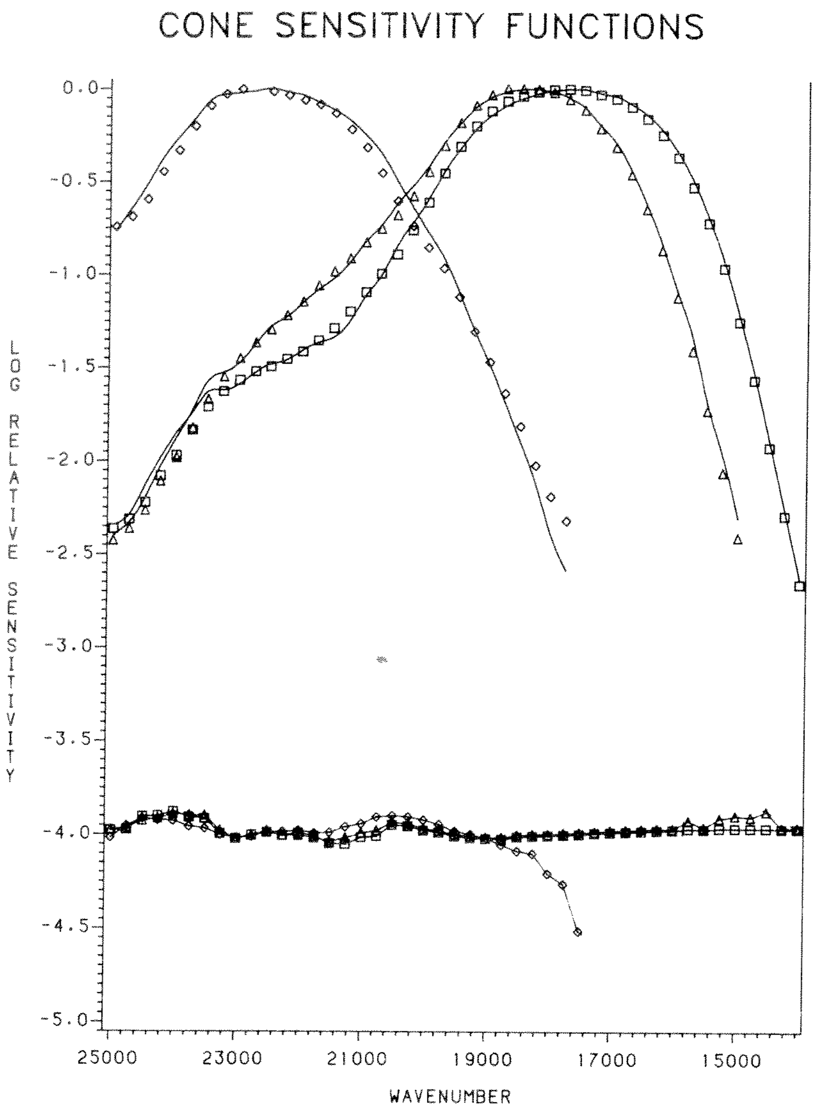

FIG. 4 The top panel shows the Smith-Pokorny (1975) fundamentals as solid lines and the fundamentals derived from the CIE observer as symbols. The bottom panel shows the difference between the two sets of log relative sensitivities centered at -4.0. Symbols: □, R; △, G; ◇, B.

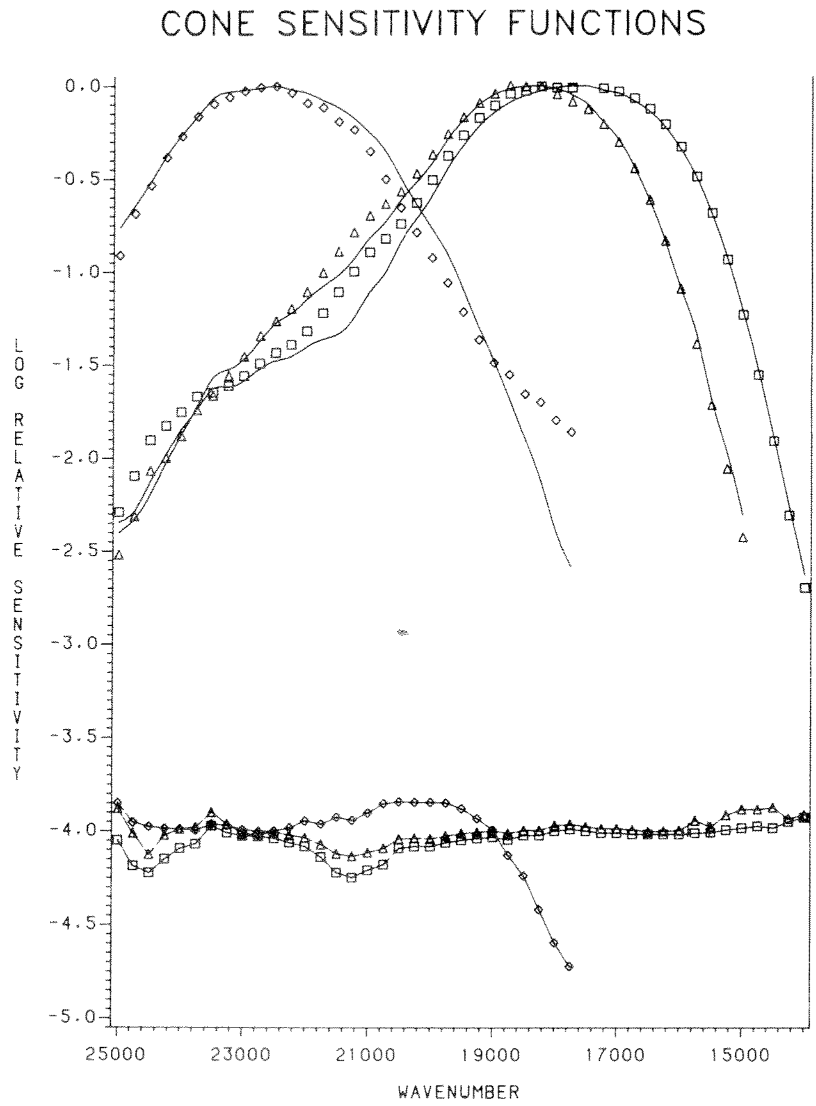

FIG. 5 Comparison of Smith-Pokorny (1975) fundamentals [solid lines] and the fundamentals derived from Stiles average observer [symbols]. Symbols: □, R; △, G; ◇, B.

wavenumbers. Estévez suggested that rod intrusion in the Stiles (1955) pilot data is one potential cause of such nonlinearity. He noted that similar nonlinearity was present in the original Guild (1931) and Wright (1929) data. This was smoothed in the process of deriving the CIE observer. If the B fundamental is truncated at 18,750 cm^{-1} (533 nm), the deviations are within 0.2 log units.

From the above analysis we conclude that the major sources of variation between the various sets of color-matching functions are the prereceptoral filters, the lens and the macular pigment. It may be noted that the derived differences fall within normal physiological variability. The macular pigment variation is relatively small. The lens values for the Judd (1951) and Stiles (1955) observers are characteristic of younger eyes than the CIE observer. We consider that the CIE (1931), the Judd (1951) and the Stiles (1955) observers are essentially equivalent for color-modelling purposes.

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